Shifting the Focus: Effective Strategies for Transitioning to a Student-Centered Approach in Mathematics Education

تحويل محور التركيز: استراتيجيات فعّالة للانتقال إلى نهج مركّز على الطالب في تعليم الرياضيات

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Abstract

Mathematics teaching and learning has been a central issue in research. Traditionally, teaching mathematics focused on a teacher-centered model. However, over time, there has been a shift in educational theory and practice, highlighting the shortcomings of this conventional approach (Catubig, 2023). As educational research evolves and teaching approaches change, it's essential for mathematics teachers to stay aligned with these shifts to ensure their students are prepared for the demands of the 21st century. This paper aims to review the literature on effective methods that support mathematics teachers in implementing a student-centered approach. The researcher primarily focuses on simple and suitable strategies for beginner teachers in their transition toward a student-centered teaching model, in particular conceptual teaching of concepts and some cooperative learning strategies that can be applied in mathematics classrooms.

Key Words: Student-centered approach, teacher-centered approach, cooperative learning techniques, Kagan Structures, 21st century skills, constructivism, conceptual teaching

الملخص

لقد كانت طرق تدريس الرياضيات وتعلمها قضية محوريّة في البحث التربوي. تقليديًا، كان تدريس الرياضيات يرتكز على نموذج يعتمد على المعلم. ومع ذلك، مع مرور الوقت،

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حدث تحول في النظريّة والممارسة التّعليميّة، ما سلط الضوء على قصور هذا النهج التقليدي (كاتوبيغ، 2023). مع تطور البحث التربوي وتغير أساليب التدريس، من الضروري أن يبقى معلمو الرياضيات متماشين مع هذه التحولات لضمان استعداد طلابهم لمتطلبات القرن الواحد والعشرين. يهدف هذا البحث إلى استعراض بعض الأدبيات المتعلقة بالأساليب الفعّالة التي تدعم معلمي الرياضيات في تنفيذ النهج القائم على الطالب. يركز الباحث بشكل أساسي على استراتيجيات بسيطة ومناسبة للمعلمين المبتدئين في انتقالهم نحو نموذج التدريس القائم على الطالب، وبشكل خاص تدريس المفاهيم والتعليم التعاوني الذي يمكن تطبيقه في صفوف الرياضيات.

الكلمات المفاتيح: نهج مركّز على الطالب، نهج مركّز على المعلّم، تقنيات التعلّم التعاوني، استراجيات كايغن، مهارات القرن الحادي و العشرين، البنّائيّة، التعليم المفاهيمي

Introduction

Mathematics teaching and learning has been a central issue in research. The process of learning mathematics is a complex one as it demands various conceptual perspectives and a comprehensive set of data to fully grasp the process (Wilkinson et al., 2018). Traditionally, teaching mathematics focused on a teacher-centered model. However, over time, there has been a shift in educational theory and practice, highlighting the shortcomings of this conventional approach (Catubig, 2023). Various countries have acknowledged the significance of mathematics and created their own strategies for effectively teaching it in schools (Council of the European Union, 2018; OECD, 2019).

Policymakers stress that mathematics instruction must adapt to meet the demands of the 21st century (Council of the European Union, 2018; OECD, 2019). The 21st century skills are the skills required for success in education and the workforce in today's economy (van Laar et al., 2020). These skills are classified into three types (Partnership for 21st Century Skills, 2007): learning skills, literacy skills and life skills. The learning skills include creativity and innovation, critical thinking and problem-solving as well as communication and collaboration. Literacy skills include information literacy, media literacy and ICT literacy. Life skills include flexibility and adaptability, initiative and self-direction, social and cross-cultural skills, productivity and accountability as well as leadership and responsibility.

In a teacher-centered classroom, students' engagement is minimized where the teacher plays a dominant role in explaining the content, modeling solving techniques and evaluating student performance. This approach limits students' critical thinking and meaningful understanding of the topics. As a result, several researches has been investigating other approaches that enhance active student engagement in their learning process (Catubig, 2023) and prepare students with the needed skills for today's world.

Student-centered approach involves a shift from the teacher being the primary source of knowledge to students actively participating in building their own understanding of mathematical concepts where they develop a holistic and meaningful experience by investigating, questioning and collaborating (Kilag et al., 2023). Student-centered learning is an approach that places students at the heart of the learning process. It involves students preparing for lessons ahead of time, actively engaging during class, and working together to reach shared academic objectives. In this approach, students tend to be more motivated and develop essential skills like critical thinking and problem-solving (Zain et al., 2012; Zakaria & Iksan, 2007; Johnson et al., 2009; Froyd & Simpson, 2010). In a student-centered classroom, teachers play a crucial role in guiding learning in the right direction and ensuring that the desired learning outcomes are achieved. While teachers are often seen as facilitators, their level of involvement varies based on the students' age and maturity, as well as the specific student-centered strategies they use. (Kuok Ho, 2023).

The student-centered approach is grounded in constructivism, where learners create meaning to their learning by connecting new information to their prior knowledge (Emaliana, 2017). Shah (2019) explained that constructivism shifts students from being passive receivers of information to active contributors in their learning journey. With guidance from the teacher, students actively build their own understanding, rather than simply absorbing knowledge from the teacher or textbooks. As a facilitator, the teacher's role is to present the material in a way that matches the student's current level of understanding. In a student-centered math class, students collaboratively learn where they work together to solve problems, explain their thinking and express themselves and coach one another (Kilag et. al, 2022). Collaboration in a mathematics classroom enhances students' understanding of and confidence in what

is being learnt (MacMath et al., 2009). Such classrooms should be a safe and positive place where students explore and apply meaningful mathematics: "Meaningful mathematics takes place in K to 12 classrooms that support students as they investigate, represent and connect mathematical ideas through discussion in the context of problem solving" (Suurtamm et al., 2015). When students take an active role in their learning, they develop ownership over their education, which boosts their motivation (Ntoumanis, 2001).

Statement of the Problem

Mathematics plays a crucial role in developing critical thinking, analytical, problem-solving, and logical reasoning skills (Smith, 2004). However, the problem is that mathematics classes are usually characterized by drills and procedural understanding (Protheroe, 2007). Mathematics has been taught in the traditional way where the teacher is the central figure, delivering lectures and leading activities, while students remain largely passive recipients of information (O'Neill & McMahon, 2005). As educational research evolves and teaching approaches change, it's essential for mathematics teachers to stay aligned with these shifts to ensure their students are prepared for the demands of the 21st century. To do so, teachers require ongoing training, guidance and support. This leads to an important research question: What are some effective mathematical instructional strategies that can equip students with the skills they need for the future? From this, another related question emerges: What teaching techniques can help teachers shift their mathematics classroom from being teacher-centered to become student-centered?

Purpose of the Study

This paper aims to review the literature on effective methods that support mathematics teachers in implementing a student-centered approach. The researcher has been teaching mathematics since 2013, initially using the teacher-centered approach before transitioning to a student-centered model. In addition to her teaching role, she has been involved in training educators to shift toward student-centered environments. Personally, the strategies discussed in this research were the first to be implemented in her own classrooms, resulting in a significant shift toward a more student-centered approach. This is why she chose to present them in this paper as they are simple and suitable for beginner teachers in their transition

toward a student-centered teaching model. The researcher primarily focuses on conceptual teaching of concepts and some cooperative learning strategies that can be applied in mathematics classrooms.

Significance and Rationale of the study

This paper presents effective instructional strategies from the literature that assist mathematics teachers in teaching, improving their teaching practices. It highlights methods that facilitate the transition from traditional teacher-centered classrooms to more dynamic, student-centered learning environments. It also outlines the implementation of collaborative learning techniques that create a more interactive and inclusive classroom atmosphere. Finally, it offers practical tips and insights for implementing each strategy, drawn from the researcher's experience in mathematics education, ensuring that teachers are equipped with realistic, actionable guidance to enhance their classroom practices.

Methodology

The researcher revised the literature to present some most effective mathematics teaching strategies that equip learners with strong foundations in today's evolving world. She focused on the importance of teaching mathematics conceptually while fostering a student-centered environment through cooperative learning techniques, all aimed at equipping students with 21st century skills. Almulla (2020) explains that meaningful learning experiences involve engaging students in discursive and disciplinary activities, rather than having them simply be passive recipients.

Conceptual Teaching of Mathematics

Procedural understanding refers to the ability to apply steps or strategies to solve various problems (**National Council of Teachers of Mathematics**, n.d.), whereas conceptual understanding involves grasping abstract concepts. Lawson (2007) wrote that students develop procedural understanding for mathematical concepts rather than conceptual.

Sinay and Nahornick (2016) explain that an effective teaching strategy for all classes is to teach mathematics for conceptual understanding as students need to understand the concepts behind procedures. They added that to enhance students' conceptual learning of mathematics, concepts behind procedures should be explained before or during the instruction of procedures (**National Council of Teachers of**

Mathematics, n.d), students should be encouraged to link procedures to the fundamental concepts behind them (Wathall, 2016), students should be asked to justify their methods and provide self- explanation (**National Council of Teachers of Mathematics**, n.d, Rittle-Johnson & Schnieder, 2014), students should be able to asses different methods of solving and incorrect ones, and finally students should be given enough time to explore unfamiliar problems before instruction (Rittle-Johnson & Schnieder, 2014).

In their study, Vargas-Hernández and Vargas-González (2022) explained that based on Dahar (2011), meaningful learning is aligned with constructivism as it emphasizes that students build knowledge through experiences by connecting new information to what they already know. This process helps them discover new concepts, solve problems, and apply knowledge to different situations; evident educational progress occurs when students link new information to their existing knowledge, enriching their cognitive framework. The core concept of constructivism is that learning is a process of construction, where learners develop new knowledge by building on what they have already learned (Phillips, 1995). That's why it is very important that teachers allow students to link procedures to underlying principles in an attempt to help students conceptually learn a certain concept rather than memorizing steps to apply it.

In conclusion, transitioning to a student-centered classroom requires a shift from focusing on procedural learning to fostering conceptual understanding. By emphasizing the connection between concepts and procedures, encouraging students to justify their thinking, and providing opportunities for exploration and reflection, teachers can create a learning environment that nurtures deeper, more meaningful learning. The key here is that teachers allow students to engage in activities to explore these connections while guiding them through the process; cooperative learning techniques can help structuring conceptual teaching and making it more effective. This approach not only enhances mathematical understanding but also empowers students to become active participants in their own learning process creating a student-centered environment.

Student-Centered Teaching Strategies

Other terms for student-centered strategies include cooperative learning strategies, learner-centered strategies and peer-led team learning strategies

(Zain et al., 2012). These strategies give students opportunities to work collaboratively in building their knowledge instead of working on their own, the thing that enhances their content understanding. In addition to that, cooperative learning techniques can be easily adapted depending on the age group, thus they can be applied in any class (Kane, 2018). When using cooperative learning strategies, teachers have a main role in structuring the groups and identifying students' tasks with clear instruction to help them know what is expected from them. In addition to that, teachers should encourage, support and facilitate students' interactions (Gillies, 2016)

Cooperative learning techniques are not simply group work; they require several components to be successful (Kane, 2018). To implement such strategies effectively, teachers should communicate expectations to students, establish routines, model steps and review the strategies often (Gregory, 2016). Following are some useful cooperative learning strategies that can be applied in any mathematics class and even for teachers that are newly following a student-centered approach.

Math Talk

Math Talk is a structured approach to classroom dialogue in mathematics, aimed at building knowledge and understanding (Hufferd-Ackles et al., 2004). The steps for a Math Talk, that is also commonly known as Number Talks and Math Talk Communities, are (Coulter, 2021):

The teacher poses a problem.

Students are given time to solve.

Students share their answers and discuss. (This will be elaborated in the coming paragraphs.)

Correct answers are presented to students.

The aim of math talks is for students who have incorrect answers to adjust their thinking and understanding by applying a strategy to reach the correct solution, while also giving other students the opportunity to share their thought processes with one another (Parrish, 2011). A math talk gives students the authority to lead math discussions where teachers become co-leaders (Huffered-Ackles et al., 2016). It encourages students to actively participate by asking questions, justifying their thinking processes and work, explaining and sharing ideas, questioning each other on their work and evaluating ideas and solutions. This collaborative process helps students co-construct their learning (Wagganer, 2015; Suurtamm et al., 2015).

Teachers support a math talk by using discussion questions, sentence

stems, examples, and requests for justification (Wagganer, 2015). To facilitate these discussions, teachers can use talk moves that vary according to their goals for these conversations (Murata et al., 2017). But it's important that teachers allow students enough time to engage in the discussion without intervening too quickly, allowing ideas to develop (Bruce, 2007). Teachers' interactions must be well prepared prior to instruction in order to ensure effective math talks (Henning et al., 2012). There are four types of talk moves that support student thinking: (a) clarifying and sharing personal thoughts, (b) focusing on the ideas of others, (c) deepening one's understanding, and (d) interacting with others' reasoning. Teachers can encourage student engagement by asking questions, giving students time to respond, rephrasing their answers for clarity, asking one student to repeat another's point to ensure active listening, and motivating students to assess and comment on their peers' ideas (Michaels & O'Connor, 2015).

Kagan Structures

Kagan is one of the resources that provides effective teaching and learning tools for cooperative learning, referred to as Kagan Structures (Kane, 2018). These structures are evidence-based teaching methods that have proven effective in enhancing both academic performance and social results. They can be applied in any classroom as early as Kindergarten (Kagan Online, n.d.) and they are content free where students can be engaged in infinite number of activities (Kagan, 2000). The key principles for a successful Kagan Structure are designing learning tasks that encourage teamwork, holding students accountable for their individual roles, ensuring equitable student participation and assuring the engagement of many students at once (Kagan, 2008).

Kagan (2000) explains that these strategies implement four basic principles, The PIES principles: Positive Interdependence, Individual Accountability, Equal Participation, and Simultaneous Interaction. Positive interdependence is when students have common goals where one person›s success benefits another, creating a sense of unity and shared purpose. Individual Accountability is by assigning every student an individual role and holding them accountable for it. Equal Participation is because all students are taking turns to finish the required task. Simultaneous Interaction is implemented since many students are engaged at once.

Clowes (2011) presents the essential 5 Kagan Structures to be used by any teacher as a starting point. These are RallyRobin, Timed Pair Share, RoundRobin, RallyCoach and Stand Up, Hand Up, Pair Up.

The following structure function table shows the categories that each of the essential 5 fit in (Clowes, 2011). For example, Timed Pair Share can be used for Teambuilding where students get to know one another, respect, value and like their teammates. Also it enhances knowledge-building where students interact to review or memorize useful material. In addition to enhancing communication and social skills, Timed Pair Share can also be used for practicing procedures, processing and presenting information and developing thinking skills.

Structure Functions	Classbuilding	Teambuilding	Social Skills	Communication Skills	Decision Making	Knowledgebuilding	Procedure Learning	Processing Info	Thinking Skills	Presenting Info
	Interpersonal					Academic				
RallyRobin			★	★		★	★	★	★	★
Timed Pair Share		★	★	★		★	★	★	★	★
RoundRobin			★	★	★		★	★	★	★	★
RallyCoach				★	★		★	★		★	
Stand Up, Hand Up, Pair Up	★			★	★		★	★	★	★	★

Figure 1: Structure Functions Table for the Essential 5 Kagan Structures

To conduct a well-structured Kagan technique, it is very essential to consider some small and simple details. A good tip for any Kagan strategy involving pairs is to identify the student that should start sharing. Teachers can use cues for this purpose: physical characteristics without using sensitive ones as weight, clothing without using judgement calls such as cuter outfit and "about me" as well as many others. The following figure shows suggestions for each category (Kagan et al., 2015). These cues can also be useful in any other non-Kagan cooperative learning strategy for pair work. Another helpful tip is running a timer to successfully pace the class. Finally, during the implementation of these strategies, teachers should be actively circulating among the students to listen to their conversations and make sure that everyone is on track and doing their part.

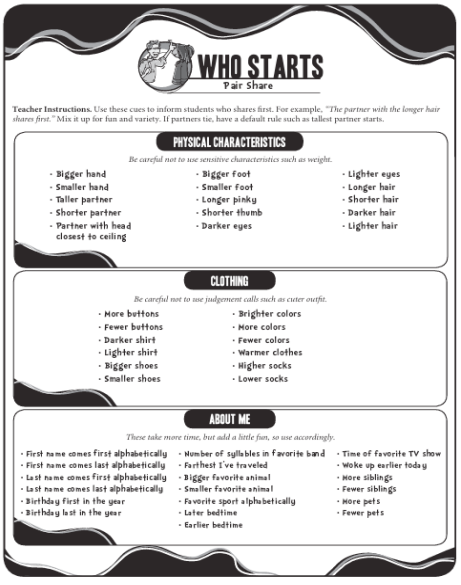

Figure 2: Cues for Choosing who Starts Pair Share

Rally Robin

Kagan (2008) explains that students should create an oral list in a Rally Robin. In this technique, every student gives multiple answers.

The steps are:

The teacher poses a question.

Partners repeatedly take turns to give one answer each time.

Here is a poster for Rally Robin (Clowes, 2011)

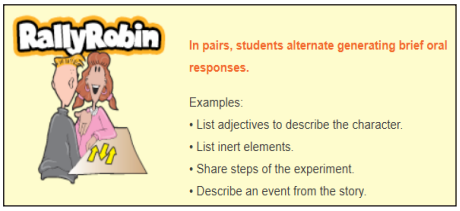

Figure 3: Rally Robin Kagan Structure

In mathematics, this strategy can be useful to revise, recall and memorize material where students are asked to list properties and share methods to prove that a quadrilateral is a square, for example. It can be used in the beginning of the session or in its end as a wrap up for what was learnt. It is a short and easy activity that engages every student in the learning process instead of having the teacher recalling the information themselves or calling certain students to do that.

Timed Pair Share

This strategy engages students in pairs to elaborate responses where students share equally for a specified time. The steps include (Kagan, 2015):

The teacher announces the topic or poses a question.

Think time is provided.

The teacher selects the partner who should start sharing for a certain time while the other is listening. This can be called partner A.

When partner A finishes, partner B shares for a specified time while the other listens.

When both partners finish, they can both raise a hand to signal completed work.

Clowes (2011) presented a poster for Timed Pair Share.

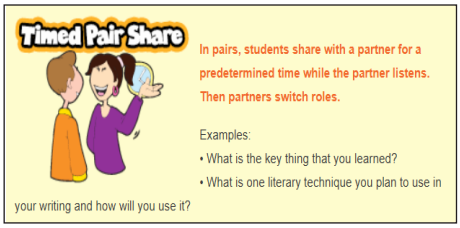

Figure 4: Timed Pair Share Kagan Structure

In a mathematics classroom, this technique can be used to have students reflect on their learning and share what they know about a concept or what they would like to learn. It is also effective in solving exercises with similar parts where each partner is assigned a part to work on during individual think time and then they share their work.

Round Robin

This strategy engages students in groups to orally answer a posed question. The steps for Round Robin include (RebelTech, n.d.):

The teacher poses a question or demonstrates a problem.

Individual think time is given.

Select a partner to start while others listen.

Each student will have a specified time to share their response taking turns clockwise or anticlockwise.

This is a poster for Round Robin (Clowes, 2011):

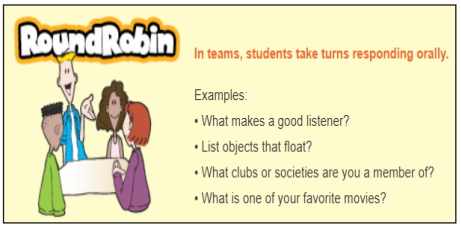

Figure 5: Round Robin Kagan Structure

In a mathematics classroom, this strategy can be helpful for brainstorming ideas where students are given equal chances to generate ideas on a specific topic. For example, teachers can benefit from it to have students practice multiplication table where each student is responsible for the multiples of a given number.

Rally Coach

In this technique, students should be paired according to their achievement levels. In each pair, the partners should be within one range of their capability level. For example, students with high levels are paired with medium-high levels and those with medium low levels can be paired with medium high or low level partner (Kagan, 2009; Stewart 2015).

The steps for a Rally Coach are (Kagan, 2009; Stewart 2015):

The teacher poses a question or a set of questions.

The partner with a higher achievement level is asked to start solving for a specified time while the other is watching, coaching and praising. It is very important that students don't this categorization. You can simply name them purposefully as partners A (higher level) and B and then ask partners A to start.

The partner that coached should sign their initials to show agreement for the other partner's work.

Partners switch roles

Clowes (2011) presents a poster for Rally Coach:

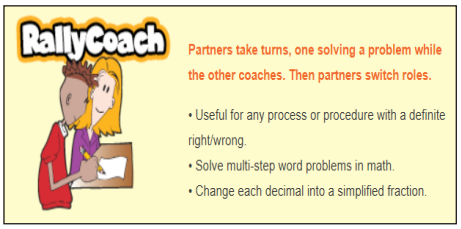

Figure 6: Rally Coach Kagan Structure

This technique is very helpful in a mathematics class. It is effective to

practice solving procedures where the teacher can present a set of exercises for students to solve including but not limited to factoring, finding the derivative function, solving equations, simplifying fractions and expressions and many other different concepts.

A good tip for Rally Coach is to apply it when students are almost close to the mastery of the concept. Another tip is to provide students with guiding prompts or questions to help coaches in their work.

Stand Up, Hand Up, Pair Up

This is a grouping structure that has no academic focus. Like other grouping structures, it is designed to efficiently move students into pairs. After grouping students, an interaction structure, such as Rally Robin, Timed Pair share, Quiz-Quiz Trade (it is not discussed in this paper) and many others, can be used to process the content of the lesson. The steps are very simple (Kagan, n.d.):

Students stand up and raise one hand high.

Students move around to find other students with their hands up.

Students give each other a high-five.

Pairs are formed.

Here is a poster for this technique by Clowes (2011):

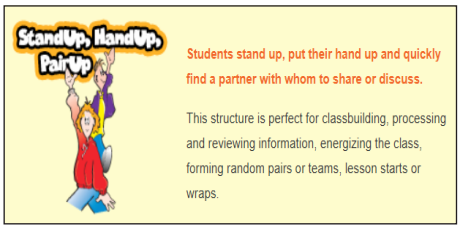

Figure 7: Stand Up, Hand Up, Pair Up Kagan Structure

In a mathematics classroom, this strategy can effectively foster a dynamic learning environment that actively engages students. When energy drops in a long math lecture, or a very difficult or easy one, it is time for students to energize and move. When students are encouraged to move, their attention and focus improve significantly compared to sitting still. To maximize its impact, it's helpful to time the activity, keep it organized, and use it repeatedly within an activity, allowing students to interact with different partners to share with and learn from. This approach is particularly beneficial for reviewing math content, such as geometric properties, or share prior knowledge about a certain topic. A good idea is to also use it for peer feedback on classwork, where students can compare answers and discuss their methods."

Conclusion

It is what and how we teach that play a crucial role in student learning

outcomes. Educators must keep the analytical and interpretive nature of mathematics, moving away from an emphasis on memorizing steps and encouraging deeper understanding. It is the role of the teacher to teach conceptually and help students better understand mathematical concepts and make connections between them. Students need to cultivate a deep comprehension by establishing strong foundational mathematical concepts and making connections between them (Faulkenberry, 2003). To enhance this conceptual learning, the implementation of constructivist approaches in the classroom is vital, as they encourage active engagement and support the development of meaningful connections between concepts. In constructivist teaching approaches, cooperative learning techniques are frequently used as when students collaborate to build their knowledge, they learn better (Slavin, 2014; Ndebil & Ali, 2024).

Our teaching style and how we deliver content play a crucial role in making a concept easier or more challenging, as well as in sparking students' interest and curiosity in the subject. Kagan (2000) explains that teachers often use structures in the classroom, whether consciously or not. One common structure is the Whole Class Question-Answer format, which is widely used across the globe. However, despite its prevalence, this approach is inefficient for achieving many of the key educational goals.

Comparing the learning environment of a traditional math class with that of a student-centered math class highlights why cooperative learning techniques lead to better learning outcomes. In a traditional math class, the teacher or one student solves problems on the board while others observe. It's unclear whether they are paying attention or simply daydreaming. Afterwards, they are asked to copy the work from the board, and even if they are focused, it may be out of fear of consequences rather than genuine interest in the subject. In contrast, a student-centered class creates a dynamic environment where every student is actively involved in the learning process. Instead of just one student solving problems while others watch, all students engage in the thinking process and collaborate with peers, providing opportunities for coaching, explanation, and discussion. This approach fosters a safe space for students to make mistakes, as it is not a "one-student show" where the focus is on who can get the answer right. Mistakes are seen as valuable learning opportunities, and this helps students

feel more confident in their abilities and encourages them to take risks in their learning. This environment naturally increases student interest in the explored topic and in mathematics, in general.

Success in schools depends on quality instruction which is directly related to actively engaging students (Kagan Online, n.d.). Cooperative learning techniques are methodologies in which students with different academic levels work collaboratively together to achieve a common goal (Bruner, 1985). Such strategies promote positive attitudes about mathematics (Leikin & Zaslavsky, 1997) and enhance students' academic performance (Artzt,1999). They also help in enhancing students' confidence and social skills (Kane, 2018). Kagan (2008) explained that the benefit of using cooperative learning structures in teaching is that it allows all students to engage actively, providing each student with multiple opportunities to answer within the same time it would normally take for a teacher to call on just two or three students. In traditional instruction, where only one student responds at a time, it would take about an hour to have each student speak for a minute, with limited participation from lower-achieving students. In contrast, with student-centered tools, every student responds, giving everyone equal opportunities for practice, which increases engagement, interest in the material, and achievement. The activities for these techniques are structured and students are held accountable for their own work and for the overall performance of the group (Wyman, 2018).

Cooperative learning strategies significantly enhance student engagement, leading to improved learning outcomes. However, the key to success lies in carefully structuring the activities and providing clear, concise instructions to minimize confusion and avoid chaos. Not every structure is cooperative or student-centered neither does every structure reach core learning outcomes. The principles of such strategies should be carefully respected. To ensure promising results, both

Eurasia Journal of Mathematics, Science & Technology Education, 3(1), 35–students and teachers should be properly trained to implement these methods.

References

1-Almulla, M. A. (2020). The effectiveness of the project-based learning (PBL) approach as a way to engage students in learning. *SAGE Open, 10*(3). https://doi.org/10.1177215824402093870 2/

2-Artzt, A. (1999). Cooperative learning in mathematics teacher education. *Mathematics Teacher, 92*(1), 11–17.

3-Bruce, C. D. (2007). Student interaction in the math classroom: Stealing ideas or building understanding. *What Works? Research into Practice*. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Bruce.pdf

4-Bruner, J. S. (1985). Vygotsky: A historical and conceptual perspective. In J. V. Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives*. Cambridge University Press. https://www.scirp.org/(S(351jmbntvnsjt1aadkposzje))/reference/ReferencesPapers.aspx?ReferenceID=1416340

5-Catubig, M. B. (2023). *Exploring student-centered approaches in mathematics education: A qualitative case study*. *Multijournals*, 1(3). https://orcid.org/00097575-5716-0006-

6-Clowes, G. (2011). *The essential 5: A starting point for Kagan cooperative learning*. *Kagan Online Magazine, Spring*. Kagan Publishing. https://www.kaganonline.com

7-Coulter, R. (2021). *Math talks impact on students in ST Math* (Master's thesis). Northwestern College. https://nwcommons.nwciowa.edu/education_masters/322

8-Council of the European Union. (2018). *Council recommendation of 22 May 2018 on key competences for lifelong learning* (ST/90092018//INIT). https://eur-lex.europa.eu/legalcontent/EN/TXT/?uri=uriserv%3AOJ.C_.2018.189.01.0001.01.ENG&toc=OJ%3AC%3A20183%A1893%ATOC

9-Dahar, R. W. (2011). *Teori-teori belajar dan pembelajaran*. Erlangga.

10-Emaliana, I. (2017). Teacher-centered or student-centered learning approach to promote learning? *Jurnal Sosial Humaniora, 10*, 59–70.

11-Froyd, J., & Simpson, N. (2010). *Student-centered learning: Addressing faculty questions about student-centered learning*. http://ccliconference.org/files/201003//Froyd_Stu-CentredLearning.pdf

12-Faulkenberry, E. E. D. (2003). *Secondary mathematics pre-service teachers' conceptions of rational numbers* (Unpublished doctoral dissertation). Oklahoma State University.

13-Gillies, R. M. (2016). Cooperative learning: Review of research and practice. *Australian Journal of Teacher Education, 41*(3). https://doi.org/10.14221/ajte.2016v41n3.3

14-Gregory, G. (2016). *Teacher as activator of learning* (pp. 99–121). SAGE Publications.

15-Haara, F. O., Bolstad, O. H., & Jenssen, E. S. (2017). Research on mathematical literacy in schools: Aim, approach, and attention. *European Journal of Science and Mathematics Education, 5*, 285–313. https://doi.org/10.30935/scimath/9512

16-Henning, J. E., McKeny, T., Foley, G. D., & Balong, M. (2012). Mathematics discussions by design: Creating opportunities for purposeful participation. *Journal of Teacher Education, 15*(5), 453–479.

17-Hufferd-Ackles, K., Fuson, K. C., & Gamoran Sherin, M. (2004). Describing levels and components of a Math-Talk Learning Community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

18-Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. B. (2016). Describing levels and components of a math-talk learning community. In E. A. Silver & P. A. Kenney (Eds.), *Lessons learned from research*. National Council of Teachers of Mathematics.

19-Johnson, A., Kimball, R., Melendez, B., Myers, L., Rhea, K., & Travis, B. (2009). Breaking with tradition: Preparing faculty to teach in a student-centered or problem-solving environment. *Primus: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 19*(2)

20-Kagan Publishing & Professional Development. (n.d.). *Kagan online*. https://www.kaganonline.com/

21-Kagan, S. (2000). *Kagan structures – Not one more program, a better way to teach any program*. Kagan Publishing. Kagan Online Magazine. https://www.kaganonline.com

22-Kagan, S. (2008). *Kagan structures simply put*. *Kagan Online Magazine*. https://www.kaganonline.com

23-Kagan, S., & Kagan, M. (2009). *Kagan cooperative learning*. Kagan Publishing.

24-Kagan, S., Kagan, M., & Kagan, L. (2015). *59 Kagan structures: Proven engagement structures*. Kagan Publishing. https://www.KaganOnline.co

25-Kagan, S. (n.d.). *Breakouts to energize brains and boost achievement*. Kagan Online. https://www.kaganonline.com/free_articles/dr_spencer_kagan/374/Breakouts-To-Energize-Brains-and-Boost-Achievement,2

26-Kane, L. (2018). *Cooperative learning in math education* (Master's thesis). Northwestern College. NWCommons. https://nwcommons.nwciowa.edu/education_masters

27-Kilag, O. K. T., Pasigui, R. E., Malbas, M. H., Manire, E. A., Piala, M. C., Araña, A. M. M., & Sasan, J. M. (2023). Preferred educational leaders: Character and skills. *European Journal of Higher Education and Academic Advancement, 1*(2), 50–56.

28-Kuok Ho, D. T. (2023). Student-centered approach in teaching and learning: What does it really mean? *Acta Pedagogia Asiana, 2*, 72–83. https://doi.org/10.53623/apga.v2i2.218

29-Lawson, A. (2007). Learning mathematics vs. following "rules": The value of student-generated methods. *What Works? Research into Practice*. Student Achievement Division.

30-Leikin, R., & Zaslavsky, O. (1997). Facilitating student interactions in mathematics in a cooperative learning setting. *Journal for Research in Mathematics Education, 28*(3), 331–354. https://doi.org/10.2307749784/

31-MacMath, S., Wallace, J., & Xiaohong, C. (2009). Problem-based learning in mathematics: A tool for developing students' conceptual knowledge. *What Works? Research into Practice*. *Research Monograph* (No. 22). The Literacy and Numeracy Secretariat.

32-Michaels, S., & O'Connor, C. (2015). Conceptualizing talk moves as tools: Professional development approaches for academically productive discussion. *In M. J. Smith, A. L. S. Wilson, & J. S. V. McKinney (Eds.), Socializing intelligence through talk and dialogue* (pp. 347–362). *National Association for the Education of Young Children*.

33-Murata, A., Siker, J., Kang, B., Baldinger, E. M., Kim, H.-J., Scott, M., & Lanouette, K. (2017). Math talk and student strategy trajectories: The case of two first-grade classrooms. *Cognition and Instruction, 35*(4), 290–316. https://doi.org/10.108007370008.2017.1362408/

34-National Council of Teachers of Mathematics. (n.d.). What is procedural fluency, and how do we help students develop it? *A position of the National Council of Teachers of Mathematics*. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics

35-Ndebil, M., & Ali, C. (2024). Cooperative learning as a strategy of improving mathematics performance and attitudes. *International Journal of Educational Innovation and Research, 3*(1), 62–74. https://doi.org/10.31949/ijeir.v3i1.7163

36-Ntoumanis, N. (2001). A self-determination approach to the understanding of motivation in physical education. *British Journal of Educational Psychology, 71*(2), 225–242.

37-OECD. (2019). *PISA 2018 assessment and analytical framework*. OECD Publishing.

38-O'Neill, G., & McMahon, T. (2005). Student-centered learning: What does it mean for students and lecturers? *AISHE-J: The All Ireland Journal of Teaching and Learning in Higher Education, 1*(1), 27–36. http://www.aishe.org/readings/20051-/

39-Parrish, S. D. (2011). Number talks build numerical reasoning. *Teaching Children Mathematics, 18*(3), 198–206. https://doi.org/10.5951/teacchilmath.18.3.0198

40-Partnership for 21st Century Skills. (2007). *Framework for 21st-century learning*. http://www.p21.org/documents/P21_Framework_Definitions.pdf

41-Phillips, D. C. (1995). The good, the bad, and the ugly: The many faces of constructivism. *Educational Researcher, 24*(7), 5–12.

42-Protheroe, N. (2007). What does good math instruction look like? *Principal, 87*(1), 51–54.

43-RebelTech, L. (n.d.). *Kagan structure: RoundRobin*. Fountain Middle School Teacher's Programs and Strategies Website. https://fmsteacherspro gramsandstrategieswebsite.wordpress.com/kagan-structure-roundrobin/

44-Rittle-Johnson, B., & Schneider, M. (2014). Developing conceptual and procedural knowledge of mathematics. In *Oxford handbook of numerical cognition* (pp. 359–373). Oxford University Press.

45-Shah, R. K. (2019). Effective constructivist teaching learning in the classroom. *Shanlax International Journal of Education, 7*(4), 1–13. https://doi.org/10.34293/education.v7i4.600

46-Sinay, E., & Nahornick, A. (2016). Teaching and learning mathematics research series I: Effective instructional strategies. *Research Report No. 1608-17/*. Toronto District School Board.

47-Slavin, R. E. (2014). *Educational psychology: Theory and practice*. Pearson.

48-Smith, A. (2004). *Making mathematics count: The report of Professor Adrian Smith's inquiry into post-14 mathematics education*. The Stationery Office. https://scholar.google.com

49-Stewart, J. M. (2015). *The implementation of two cooperative learning structures, Rally Coach and Teams-Games-Tournaments, in high school chemistry courses* (Master's thesis). Louisiana State University. https://repository.lsu.edu/gradschool_theses/1684

50-Suurtamm, C., Quigley, B., & Lazarus, J. (2015). Making space for students to think mathematically. *What Works? Research into Practice*. *Research Monograph* (No. 59). Student Achievement Division.

51-van Laar, E., van Deursen, A. J. A. M., van Dijk, J. A. G. M., & de Haan, J. (2020). Determinants of 21st-century skills and 21st-century digital skills for workers: A systematic literature review. *SAGE Open, 10*(1), 1–14. https://doi.org/10.1177215824401990017 6/

52-Vargas-Hernández, J., & Vargas-González, O. (2022). Strategies for meaningful learning in higher education. *Journal of Research in Instructional, 2*(1), 47–64. https://doi.org/10.30862/jri.v2i1.41

53-Wagganer, E. L. (2015). Creating math talk communities. *Teaching Children Mathematics, 22*(4), 248–254.

54-Wathall, J. T. (2016). *Concept-based mathematics: Teaching for deep understanding in secondary classrooms*. Corwin Press.

55-Wilkinson, L. C., Bailey, A. L., & Maher, C. A. (2018). Students' mathematical reasoning, communication, and

language representations: A video-narrative analysis. *ECNU Review of Education, 1*(3), 1–22. https://doi.org/10.30926/ecnuroe.2018010301

56-Wyman, P. J. (2018). *Academic achievement with cooperative learning using homogeneous and heterogeneous groups* (Doctoral dissertation). Liberty University. https://phd.cooperativelearning.pdf

57-Zain, S., Rasidi, F., & Zainol, I. (2012). Student-centred learning in mathematics: Constructivism in the classroom. *Journal of International Education Research (JIER), 8*(4), 319–328. https://doi.org/10.19030/jier.v8i4.7277

58-Zakaria, E., & Iksan, Z. (2007). Promoting cooperative learning in science and mathematics education: A Malaysian perspective39.